

# Electron mobility on a surface of dielectric media: influence of surface level atoms

P. D. Grigoriev\* and A.M. Dyugaev

*L. D. Landau Institute for Theoretical Physics, Chernogolovka, Moscow region, 142432, Russia*

E.V. Lebedeva

*Institute of Solid State Physics, Chernogolovka, Moscow region, 142432, Russia*

We calculate the contribution to the electron scattering rate from the surface level atoms (SLA), proposed in [A.M. Dyugaev, P.D. Grigoriev, JETP Lett. 78, 466 (2003)]. The inclusion of these states into account was sufficient to explain the long-standing puzzles in the temperature dependence of the surface tension of both He isotopes and to reach a very good agreement between theory and experiment. We calculate the contribution from these SLA to the surface electron scattering rate and explain some features in the temperature dependence of the surface electron mobility. This contribution is essential at low temperature  $T < 0.5$  when the He vapor concentration is exponentially small. For an accurate calculation of the electron mobility one also needs to consider the influence of the clamping electric field on the surface electron wave function and the temperature dependence of the He3 chemical potential.

Two-dimensional electron gas on a surface of dielectric media is wide a subject of research for several decades (for reviews see, e.g., [1, 2, 3]). The electrons are attracted to the dielectric interface by electric image forces and become localized in the direction perpendicular to the surface. The surface of liquid helium has no solid defects (like impurities, dislocations, etc.) and gives a unique chance to create an extremely pure 2D electron gas. This system simulates the solid-state 2D quantum wells without disorder. Many fundamental properties of a 2D electron gas have been studied with the help of electrons on the surfaces of liquid helium. The many-body electron effects on the surface of liquid helium are determined by the interaction between electrons and surface waves (ripplons) and by the Coulomb electron-electron (e-e) interaction screened by a substrate. The Wigner crystallization of 2D electron gas, induced by the Coulomb e-e interaction, was first observed and extensively studied on the surface of liquid helium (see [1, 2, 3] for reviews). A large number of interesting quantum objects may be experimentally realized with electrons on the liquid helium surface: the quantum rings, the 1D electron wires, the quantum dots etc. The electrons on the liquid helium surface may also serve for an experimental realization of a set of quantum bits with very long incoherence time.[4] All these quantum objects and phenomena depend in a crucial way on the structure and properties of the liquid helium surface itself.

The interface between liquid helium and vacuum is usually supposed to be sharp – the density of helium atoms falls to zero on a distance of intermolecular spacing which is much smaller than the size of the surface electron wave function. The electrons are clamped to the surface by the image forces and by external electric field. The electron wave function on the lowest energy level in  $z$ -direction without external field is given by [1, 2, 3]

$$\Psi_e(z) = 2\gamma^{3/2}z \exp(-\gamma z), \quad (1)$$

where in the absence of clamping electric field  $\gamma = m\Lambda/\hbar^2$  and  $\Lambda \equiv e^2(\epsilon - 1)/4(\epsilon + 1)$ . The dielectric constants of liquid  $^4\text{He}$  is  $\epsilon_4 = 1.0572$  and  $\gamma = (76\text{\AA})^{-1}$ , while for  $^3\text{He}$   $\epsilon_3 = 1.0428$  and  $\gamma = (101\text{\AA})^{-1}$ . Hence, the electron wave function is rather extended in  $z$ -direction, which reduces the influence of small surface ripple on the electron motion and makes the mobility of 2D electrons gas on the helium surface to be rather high. At low electron concentration  $N_e \approx 10^7/\text{cm}^2$  the mobility of electrons on the liquid helium surface at  $T = 0.1\text{K}$  reaches  $10^4\text{m}^2/\text{Vsec}$ . [5], which is about  $10^4$  times greater than the carrier mobility in heterostructures. For several decades people believed that at low enough temperature, i.e. when the concentration of He vapor is exponentially small, on the surface of liquid helium there is only one type of excitations interacting with surface electrons. These excitations are the quanta of surface waves, called ripplons. These surface waves determine the temperature dependence of the surface tension of liquid helium, as well as the mobility, cyclotron resonance line-width and other properties of surface electrons at temperature below  $\sim 0.5\text{K}$ . [1, 2, 3]

Recently, we proposed [6] that in addition to the ripplons, there is another type of surface excitations – the surface level atoms (SLA). The SLA considerably influence the temperature dependence of the surface tension of both liquid helium isotopes. Taking the SLA into account we explained the long-standing puzzles [6, 7] in this temperature dependence and reached a very good agreement (up to 0.1%) between theory and experiment on the surface tension

---

\*Electronic address: grigorev@itp.ac.ru

in a large temperature interval. This high agreement may serve as an additional proof of the existence of SLA as the new type of excitations on the surface of liquid helium.

An accurate microscopic description of this new type of excitations is a rather complicated problem that requires an investigation of many-particle interactions in liquid helium. However, one may consider SLA phenomenologically, assuming them to be similar to the quantum states of helium atoms which are localized at the liquid helium surface. The SLA may also propagate in the surface plane and have the quadratic dispersion

$$\varepsilon(k) = E_{SLA} + k^2/2M^*,$$

where  $k$  is the 2D momentum of this excitation along the surface. Both the SLA energy  $E_{SLA}$  and the effective mass  $M^*$  of the SLA differ for the two He isotopes  $^3\text{He}$  and  $^4\text{He}$ . The SLA energies  $E_{SLA}$  are intermediate between the energy of He atom in vacuum  $E_{vac}$  and the chemical potential  $\mu$  of this atom inside the liquid. If one takes the energies of He atoms in vacuum to be zero,  $E_{vac}^{He} = 0$ , the chemical potential at  $T \rightarrow 0$  is  $\mu^{He4} = -7.17K$  and  $\mu^{He3} = -2.5K$ , while the energies of these atom on the surface levels, as suggested by the temperature dependence of the surface tension,[6] are  $E_{SLA}^{He4} \approx -3.5K$  and  $E_{SLA}^{He3} \approx -2.25K$ . Therefore, at low enough temperature the concentration of SLA becomes exponentially higher than the He vapor concentration, and the influence of the SLA on the properties of surface electrons becomes more important than the influence of the He vapor.

On the other hand, there is a long-standing discrepancy between the theoretical prediction[8] for the electron mobility on the liquid helium surface and the experimental data (see Fig. 2 of Ref. [5]). According to the existing theory[8] the ratio of electron mobilities on  $^3\text{He}$  and  $^4\text{He}$  surfaces at the same concentration of helium vapor must be equal to  $\gamma_{He4}/\gamma_{He3} = 1.33$ , while the experimental lines (see Fig. 2 of Ref. [5]) even cross each other. In the present report we calculate the influence of the SLA on the mobility of 2D surface electrons and analyze if taking into account this influence helps to explain the existing discrepancy between the theory and experiment. Note that the similar surface states may also occur in other liquids and solids, as solid hydrogen or neon, raising similar questions.

Vapor atoms or SLA can be considered as point-like impurities localized at points  $r_i$ . These impurities interact with electrons via a  $\delta$ -function potential  $V_i(r) = U\delta(r - r_i)$ . Then there is no difference between transport and usual mean free time  $\tau$ , which is given by:

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \int dz N_{He}^{tot}(T, z) \int \frac{d^2\mathbf{p}' |T_{\mathbf{p}\mathbf{p}'}(z)|^2}{(2\pi\hbar)^2} \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'}), \quad (2)$$

where  $\varepsilon(\mathbf{p}) = \mathbf{p}^2/2m^*$  is electron dispersion relation,  $m^*$  is the effective electron mass,  $|\mathbf{v}_{\mathbf{p}}| = p/m^*$  is the electron velocity. The 2D matrix element of the electron scattering by helium atom

$$T_{\mathbf{p}\mathbf{p}'}(z) = |\Psi_e(z)|^2 U,$$

The integration over  $\mathbf{p}'$  in (2) cancels the delta-function and one obtains

$$\begin{aligned} \frac{1}{\tau} &= \int dz N_{He}^{tot}(T, z) \Psi_e^4(z) \frac{U^2 m}{\hbar^3} \\ &= \int dz N_{He}^{tot}(T, z) \Psi_e^4(z) \frac{A\hbar\pi}{m}, \end{aligned}$$

where  $A = m^2 U^2 / \pi \hbar^4$  is the cross section of the electron scattering on the He atom. Note that the cross section of 2D electrons is greater than 3D one [2].

The total density  $N_{He}^{tot}(T, z)$  of helium atoms as function of distance to the surface is a sum of two parts:

$$N_{He}^{tot}(T, z) = N_v(T) + n_s(T, z). \quad (3)$$

The first part  $N_{He}^{tot}(T, z)$  is the density of helium vapor. It is roughly independent of  $z$  and is given by

$$N_v = \alpha \left( \frac{M k_B T}{2\pi\hbar^2} \right)^{3/2} \exp \left( \frac{\mu^{He} - E_{vac}^{He}}{k_B T} \right), \quad (4)$$

where  $\alpha = 1$  for  $^4\text{He}$  and  $\alpha = 2$  for  $^3\text{He}$  because of the spin degeneracy. The second part  $n_s(z)$  is the density of SLA. It depends on the wave function  $\Psi_s(z)$  of an atom on the surface level:

$$N_s(T, z) = n_s(T) \Psi_s^2(z). \quad (5)$$

Electron mobility

$$\eta_e = \frac{\tau}{m} = \frac{1}{\pi \hbar A [N_v(T)I_v + n_s(T)I_s]} \quad (6)$$

where we introduced the notations

$$I_s = \int \Psi_e^4(z) \Psi_s^2(z) dz \quad (7)$$

and

$$I_v = \int \Psi_e^4(z) dz \approx \int_0^\infty dz \left[ 2\gamma^{3/2} z \exp(-\gamma z) \right]^4 = 3\gamma/8, \quad (8)$$

In the absence of the SLA we obtain

$$\eta = \frac{8}{3\pi \hbar A \gamma N_v(T)}$$

in agreement with [8].

To calculate the integral (7) one need to know the wave function  $\Psi_s^2(z)$ . An exact calculation of  $\Psi_s(z)$  is a complicated many-particle problem. To estimate the contribution of scattering on surface atoms to the electron mobility one can use an approximate wave function (1) with  $\gamma = \gamma_s = \sqrt{-2ME_{SLA}}/\hbar$ , where  $E_{SLA}$  is the energy of surface level and  $M$  is the free atom mass:  $M^{He4} = 6.7 \cdot 10^{-24}g$  and  $M^{He3} = 5.05 \cdot 10^{-24}g$ . The value  $E_s$  can be taken from the contribution of surface states to the temperature dependence of surface tension. According to Ref. [6]  $E_s^{He4} \approx 3.5K$  and  $E_s^{He3} \approx 2.25K$ , that gives  $\gamma^{He4} \approx (1.3\text{\AA})^{-1}$  and  $\gamma^{He3} \approx (1.87\text{\AA})^{-1}$ . In this approximation the integral (7) coming from SLA is

$$\begin{aligned} I_s(T) &= \int 64\gamma_e^6 \gamma_s^3 z^6 \exp[-2(\gamma_s + 2\gamma_e)z] dz \\ &= \frac{360\gamma_s^3 \gamma_e^6}{(\gamma_s + 2\gamma_e)^7}. \end{aligned} \quad (9)$$

The ratio

$$r \equiv \frac{n_s(T)I_s}{N_v(T)I_v} \approx \frac{n_s(T)}{N_v(T)} \frac{960\gamma_s^3 \gamma_e^5}{(\gamma_s + 2\gamma_e)^7} \quad (10)$$

may be less or greater than unity depending on temperature and clamping electric field. This ratio determines the role of SLA in the momentum relaxation of surface electrons. At low enough temperature, when the vapor atom density  $N_v(T)$  is negligible compared to the SLA density because of the large negative exponent in (4), the ratio (10) is  $\gg 1$ . In the opposite limit of high temperature, when  $N_v(T)$  is not negligibly small, the ratio (10) is  $\ll 1$  because of the second factor in the right hand side of Eq. (10), which contains the smallness  $(\gamma_e/\gamma_s)^4$ . The numerical calculation of the SLA wave function  $\Psi_s(z)$  assuming the van der Waals interaction between He atoms gives a slightly greater value for the ratio (10):

$$r \approx \frac{n_s(T)}{N_v(T)} \frac{2 \cdot 10^4 \gamma_s \gamma_e^5}{(\gamma_s + 2\gamma_e)^5}. \quad (11)$$

The 2D SLA density  $n_s(T)$  differs for  $^3\text{He}$  and for  $^4\text{He}$ . [6] For  $^4\text{He}$  it is given by density of states of a 2D Bose gas: with effective mass

$$\begin{aligned} n_4(T) &= \int \frac{d^2k}{(2\pi\hbar)^2} \cdot \frac{1}{\exp[(\varepsilon_4(k) - \mu_{He4})/T] - 1} \\ &= -\frac{M_4 T}{2\pi\hbar^2} \ln \left[ 1 - \exp\left(-\frac{\Delta_4}{T}\right) \right], \end{aligned}$$

where  $\Delta_4 = E_{SLA}^{He4} - \mu^{He4} \approx 3.6K$  is almost temperature independent and the SLA effective mass  $M_4 \approx 2.6 M_0^{He4}$ . For  $^3\text{He}$  the analogous calculation would gives

$$\begin{aligned} n_3(T) &= 2 \int \frac{d^2k}{(2\pi\hbar)^2} \cdot \frac{1}{\exp[(\varepsilon_3(k) - \mu_{He3})/T] + 1} \\ &= \frac{M_3 T}{\pi\hbar^2} \ln \left[ 1 + \exp\left(-\frac{\Delta_3(T)}{T}\right) \right], \end{aligned} \quad (12)$$

where

$$\Delta_3(T) = E_{SLA}^{He3}(T) - \mu^{He3}(T). \quad (13)$$

At  $T \rightarrow 0$ ,  $\Delta_3 = E_{SLA}^{He3} - \mu^{He3} \approx 0.25K$ , and  $M_3 \approx 2.25M_0^{He3}$ . The temperature dependence of  ${}^3\text{He}$  chemical potential  $\mu^{He3}(T)$  is stronger than that for  ${}^4\text{He}$  and may be essential even at low temperature.[10, 11]

To analyze the temperature dependence of the  ${}^3\text{He}$  chemical potential  $\mu^{He3}(T)$  one may apply the exact thermodynamic relation [9]

$$\mu = \bar{F} + \frac{P}{n_L}, \quad (14)$$

where  $\bar{F}$  is the free energy per one atom,  $P$  is the pressure and  $n_L$  is the liquid density. If temperature is not too high,  $P = N_v T$ , where the vapor density  $N_v(T)$  is given by Eq. (4). Since  $N_v \ll n_L$ , the second term in Eq. (14) is small and the dependence  $\mu_3(T)$  can be found using the relation between free energy and specific heat  $C_3(T)$  per one  ${}^3\text{He}$  atom,[9]

$$\mu_3(T) = \mu_3(0) + \int_0^T C_3(T_1) dT_1 - T \int_0^T \frac{C_3(T_1) dT_1}{T_1}. \quad (15)$$

Below we find the temperature dependence  $\mu_3(T)$  basing on the experimental data on  $C_3(T)$ . [12] The latter can be fitted with a high accuracy by the phenomenological formula

$$C_3(T) = 0.4T + 0.105 \frac{T}{T^2 + T_0^2}; \quad T_0 = 0.21K, \quad (16)$$

where temperature  $T$  is in Kelvins:  $[T] = K$ . A theoretical substantiation of (16) is given in Ref. [11]. From (15,16) one obtains:

$$\Delta_3(T) = 0.25 + 0.2T^2 + 0.5T \arctan\left(\frac{T}{T_0}\right) - 0.053 \ln\left(1 + \frac{T^2}{T_0^2}\right). \quad (17)$$

Unfortunately, we could not determine from Ref. [5] if the helium vapor density on Fig. 2 of Ref. [5] was experimentally measured or calculated using Eq. (4). In the latter case the temperature dependence of the chemical potential  $\mu_3(T)$  taken in Ref. [5] remains unknown. Therefore, in the comparison of the results of our calculation with experimental data we take the chemical potential  $\mu_3(T)$  to be temperature independent.

In strong clamping electric field the scattering rate of electrons increases due to an increase of the electron velocity in z-direction:  $v_{ze} \approx \hbar\gamma_e/m_e$ . [2] For the scattering on vapor atoms this increase is slower than for the scattering on SLA and ripplons. One cannot find analytically the wave function of surface electrons in the presence of both image potential and the clamping field. For an approximate study one can apply the variational method with the trial wave function (1), where  $\gamma$  is the variational parameter. One gets [1]

$$\gamma = \frac{\gamma_1}{3} \left[ \frac{\gamma_0}{\gamma_1} + \left( 1 + \sqrt{1 - \left( \frac{\gamma_0}{\gamma_1} \right)^6} \right)^{1/3} + \left( 1 - \sqrt{1 - \left( \frac{\gamma_0}{\gamma_1} \right)^6} \right)^{1/3} \right], \quad (18)$$

where  $\gamma_1 = (\gamma_0^3 + 27\gamma_F^3/2)^{1/3}$ ,  $\gamma_0 \equiv \gamma(E_\perp = 0)$ , and  $\gamma_F^3 = 3meE_\perp/2\hbar^2$ . Note that  $\gamma_e$  enters the ratio (10),(11) in the fifth power the ratio (10),(11), and in stronger clamping field the role of the scattering on SLA becomes more important. The increase of  $\gamma$  given by Eq. (18) enhances the electron scattering by He atoms and decreases the mobility of surface electrons. For the electron concentration  $n_e = 1.21 \cdot 10^7 \text{cm}^{-2}$  as in the experiment on  ${}^3\text{He}$  in Ref. [5] using  $eE_\perp = 2\pi e^2 n_e$  one obtains the value  $\gamma_F/\gamma_0 = 0.28$ , which gives  $\gamma/\gamma_0 \approx 1.02$ . This ratio changes by only 2% the contribution from He vapor to the electron scattering rate. This ratio enters in sixth power to the SLA contribution [see Eqs. (10),(11)], and according to the above consideration the clamping electric field at electron density in the experiment [5] increases by 14% the SLA contribution to the scattering rate of electron on  ${}^3\text{He}$  surface and by 5% the SLA contribution for  ${}^4\text{He}$ . This correction is small, though it may increase if more accurate study of the electron wave function in clamping field on the surface of liquid helium is performed. We leave this more accurate study for further publications and disregard this correction in what follows.

To compare the calculated mobility of electrons on  $^4\text{He}$  surface with experiment one has to take into account the electron scattering by riplons. The ripplon-limited mobility in weak clamping field is given by [1]

$$\eta_R = \frac{9\sigma\hbar^3}{m^2\Lambda^2\gamma^2T} \left[ \frac{cm}{dyn \cdot c} \right], \quad (19)$$

while in strong clamping field  $E_\perp$  it is

$$\eta_R = \frac{8\sigma\hbar}{m(eE_\perp)^2}. \quad (20)$$

The surface tension of  $^4\text{He}$   $\sigma^4 = 0.37\text{dyn/cm}$ , and in the limit of weak clamping field we get

$$\eta_R \approx \frac{3 \cdot 10^7}{T[K]} \left[ \frac{cm^2}{Vc} \right]. \quad (21)$$

The total electron mobility is roughly given by

$$\eta_{tot}^{-1} = \eta_e^{-1} + \eta_R^{-1}, \quad (22)$$

where  $\eta_e^{-1}$  is given by Eq. (6).

From Eq. (11) we obtain for  $^4\text{He}$

$$r \approx \frac{5 \cdot 10^{-4}}{\sqrt{T}} \exp\left(\frac{3.6}{T}\right).$$

$r = 1$  at  $T \approx 0.5K$ . Hence, the contribution from the scattering on SLA becomes greater than that on He vapor at  $T \lesssim 0.5K$ . At this temperature the contributions from all three scattering mechanisms are of the same order. Far from this temperature the contribution from the SLA to the electron scattering rate on the surface of  $^4\text{He}$  is usually a small correction to the total scattering rate. This correction becomes more important in strong clamping field, i.e. for higher concentration of surface electrons.

At low temperature the  $^3\text{He}$  viscosity becomes very high, and all riplons disappear in the temperature region  $T_{SF} < T < T_F$ , where  $T_{SF}$  is the transition temperature to the  $^3\text{He}$  superfluid state. Therefore, in the low temperature part of the phase diagram the contribution from ripplon to the electron scattering is much weaker than for He4, making the contribution from SLA more important. The results of the calculation of the temperature dependence of electron mobility on  $^3\text{He}$  surface together with the experimental data from [5] are shown in Fig. 1. From Fig. 1 we see that the agreement between experimental data on electron mobility on the liquid  $^3\text{He}$  surface and the theoretical calculation considerably improves if the SLA are taken into account. The agreement is better at low temperature, where the contribution of SLA is stronger than the contribution from He vapor. However, the inclusion of SLA does explain all discrepancies between theory and experiment. In particular, the experimental points show a  $\sim 2$  times lower mobility than the theory prediction. This discrepancy may come from the inaccurate value of the He atom cross section, which enters the electron scattering rate. The crossing of the experimental lines for the electron mobility  $\eta(N_g)$  as function of He vapor concentration, clearly seen in Fig. 2 of Ref. [5], also remains unexplained. A little better agreement with experiment can be reached after taking into account the modification of the surface electron wave function by clamping electric field [see Eq. (18) and discussion after this formula].

To summarize, we calculated the contribution to the electron scattering rate from the surface level atoms, proposed in Ref. [6]. These SLA explain the long-standing puzzles in the temperature dependence of the surface tension of both He isotopes and allow to reach a very good agreement between theory and experiment.[6] These surface atom states also make a contribution to the surface electron scattering rate and reduce the mobility of the surface electrons. This contribution may be essential at low temperature  $T < 0.5$  when the He vapor concentration is exponentially small. The contribution from the SLA improves an agreement between theory and experiment on surface electron mobility. However, this contribution alone is not sufficient to explain all puzzles in the temperature dependence of the surface electron mobility. For more accurate calculation of the electron mobility one needs to take into account the influence of the clamping electric field on the surface electron wave function and the temperature dependence of the He3 chemical potential.

The work was supported by RFBR grant No 06-02-16551 and by INTAS No 01-0791.

---

[1] V.B. Shikin and Yu.P. Monarkha, *Two-Dimensional Charged Systems in Helium* (in Russian), Nauka, Moscow (1989).

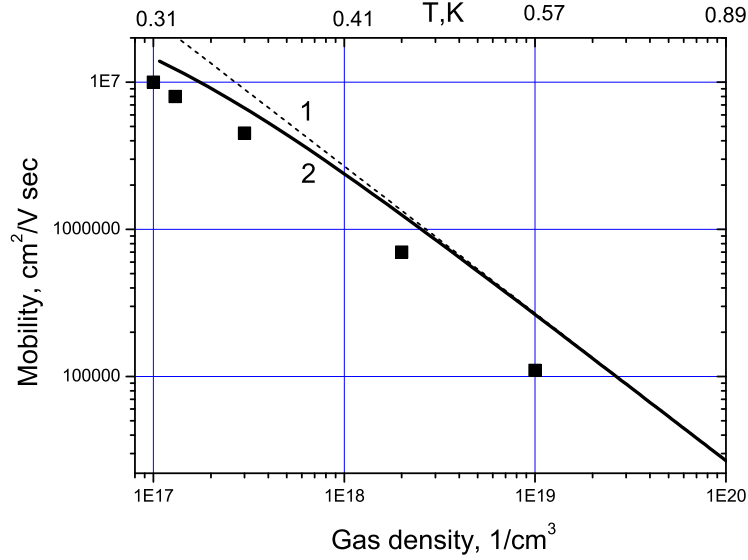


FIG. 1: The mobility of surface electrons as function of He vapor density for  $^3\text{He}$  in logarithmic scale. The dots are the experimental data from [5]. The dash line is the theoretical prediction by Saitoh.[8] The solid line is our modification of the Saitoh results where we take into account the SLA contribution to the electron scattering rate.

- [2] V.S. Edel'man, Sov. Phys. - Uspehi **130**, 676 (1980).
- [3] Y. Monarkha, K. Kono, *Two-Dimensional Coulomb Liquids and Solids*, Springer Verlag, 2004.
- [4] P.M. Platzman, M.I. Dykman, Science **284**, 1967 (1999); M. I. Dykman, P. M. Platzman, and P. Seddighrad, Phys. Rev. B **67**, 155402 (2003)
- [5] K.Shirahama, S. Ito, H.Suto and K. Kono, J. Low Temp. Phys. **101**, 439 (1995).
- [6] A.M. Dyugaev, P.D. Grigoriev, JETP Lett. **78**, 466 (2003).
- [7] M. Iino, M. Suzuki, A.J. Ikushima and Y. Okuda, J. Low Temp. Phys. 59, 291 (1985); M. Suzuki, Y. Okuda, A.J. Ikushima and M.Iino, Europhys. Lett. 5, 333 (1988); K. Matsumoto, Y. Okuda, M. Suzuki and S. Misawa, J. Low Temp. Phys. 125, 59 (2001).
- [8] M. Saitoh, J. Phys. Soc. Japan **42**, 201 (1977).
- [9] L. D. Landau and E. M. Lifshitz, Course of Theoretical Physics, Vol. 5: Statistical Physics, 3rd ed. (Nauka, Moscow, 1976; Pergamon Press, Oxford, 1980).
- [10] A.M. Dyugaev, J. Low Temp. Phys. 78, 79 (1990).
- [11] A.M. Dyugaev, Sov. Sci. Rev. A. Phys. 14, 1 (1990).
- [12] D.S. Greywall, Phys. Rev. B 27, 2747 (1983).
- [13] A. F. Andreev, Zh. Eksp. Teor. Fiz. 50, 1415 (1966) [Sov. Phys. JETP 23, 939 (1966)].